RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2018

THIRD YEAR [BATCH 2016-19]

Date : 21/12/2018 **MATHEMATICS [Honours]**

Time : 11 am – 3 pm **Paper : VI** Full Marks : 100

[Use a separate Answer Book for each Group]

Group – A

[All symbols have their usual meaning]

Answer any six questions from Question Nos. 1 to 9 : [6×5]

1. Discuss about the existence and uniqueness of the polynomial interpolation formula for (n+1) distinct points, $n \in \mathbb{N}$. [3+2]

2. Prove the following for divided differences $f\left[x_0, x_1, ...x_n\right] = \dfrac{\Delta^n f(x_0)}{n!h^n}$ for equidistant arguments, where, $x_r = x_0 + rh$, r =0,1,2.......,n, h>0 and Δ is the forward difference operator. [5]

3. Find the error in Simpson's one-third rule in the closed form solution for numerical integration. [5]

4. Certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871), using Lagrange's polynomial interpolation formula, find $\log_{10}301$. [5]

5. Determine N, the number of sub-interval from the error formula $-\dfrac{(b-a)^5}{180N^4}.f^{iv}(\xi)$, so that the composite Simpson's one third rule gives the value of the integral $\int_0^1 \dfrac{dx}{1+x}$ correct to three decimal places. [5]

6. a) Use bisection method to approximate a root of $x^3-7x^2+14x-16=0$ in the interval [0,1] accurate within tolerance of 10^{-1}.
   b) Why is the convergence, to actual root, guaranteed in case of bisection method? [3+2]

7. Define order of convergence of an iterative method for solution of algebraic and transcendental equation. Find the order of convergence of Newton-Raphson iteration method. [1+4]

8. Solve, by Gauss-Seidel iteration method, the system
$$x_1 + x_2 + 4x_3 = 9$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
up to three significant figures. [5]

9. Use Euler method to approximate the solution to the initial value problem
$$\frac{dy}{dt} = y - t^2 + 1, \quad 0 \le t \le 2$$
$$y(0) = 0.5$$
with h = 0.5 as the step in t-direction (Give answer correct upto 2 decimal place) [5]

Answer __any two__ questions from __Question Nos. 10 to 12__: [2×10]

10. a) Find the acute angle between the surfaces $z = x^2+y^2$ and $z=\left(x-\frac{1}{\sqrt{6}}\right)^2+\left(y-\frac{1}{\sqrt{6}}\right)^2$ at the point $P=\left(\frac{\sqrt{6}}{12},\frac{\sqrt{6}}{12},\frac{1}{12}\right)$. [4]

b) If f(r) is differentiable then prove that $\vec{\nabla}^2 f(r)=f^{//}(r)+\frac{2}{r}f^{/}(r)$, where $r=\sqrt{x^2+y^2+z^2}$. Hence, deduce that $\vec{\nabla}^2\left(\frac{1}{r}\right)=0$ [3+1]

c) Evaluate $\iint_S \vec{r}.d\vec{S}$ where S is the surface of a sphere of radius 'a' with centre at (0, 0, 0). [2]

11. a) Suppose $\vec{A}=6z\hat{i}+(2x+y)\hat{j}-x\hat{k}$. Using Gauss-divergence theorem evaluate $\iint_S \vec{A}.d\vec{S}$ over the entire surface bounded by the cylinder $x^2+y^2 = 9$ and the planes $x = 0$, $y = 0$, $z = 0$ and $y = 8$. [4]

b) Find the area bounded by one arch of the cycloid $x=a(t-\sin t), y=a(1-\cos t), a>0$ and $x-$axis. [3]

c) Use Stoke's theorem to evaluate $\int_C (\sin z\,dx-\cos x\,dy+\sin y\,dz)$, where C is the boundary of the rectangle: $0\le x\le\pi,\ 0\le y\le 1,\ z=3$ [3]

12. a) Let $\vec{F}=(2x+y)\hat{i}+(3y-x)\hat{j}$. Evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve in the xy plane. Consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2). [4]

b) Find the equations for the tangent plane and normal line to the surface $z=x^2+y^2$ at the point $(2,-1,5)$. [4]

c) Does there exist any differentiable vector function $\vec{V}$ such that $\vec{\nabla}\times\vec{V}=\vec{r}$. [2]

__Group - B__

Answer __any two__ questions from __Question Nos. 13 to 15__ : [2×12]

13. a) Obtain the condition so that a given straight line may be a principal axis of the material system at any point of its length and if so find the other two principal axes at that point. [6]

b) Three uniform rods AB, BC, CD are hinged freely at their ends, B and C, so as to form three sides of a square and are laid on a smooth table. The end A is struck by a horizontal blow P at right angles to AB. Show that the angular velocity of A is nineteen times that of D and that the impulsive action at B is $\frac{5P}{12}$. *[6]*

14. a) A plank of length a and of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mas $M^/$ starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M^/ a}{(M+M^/)g\sin\alpha}}$ [5]

b) A solid circular cylinder of radius 'a' rotating about its axis is placed gently with its axis horizontal on a rough plane, whose inclination to the horizon is α. Initially, the friction acts up the plane and the coefficient of friction is μ. Show that the cylinder will move upwards, if $\mu > \tan\alpha$. Also, show that the time that elapses before rolling commences is $\frac{a\Omega}{g(3\mu\cos\alpha-\sin\alpha)}$, where Ω is the initial angular velocity of the cylinder. [7]

15. a) Show that the resultant kinetic energy of a rigid body moving in two dimensions under finite forces is equal to the sum of two kinetic energies, one due to translation and the other due to rotation. [6]

b) A uniform rod of mass m and length 2a, can turn freely about a fixed end. Show that the least angular velocity with which it must be started from the lowest position, so that it may just reach the upward vertical position is $\sqrt{\frac{3g}{a}}$. Prove further that with this starting angular velocity the rod will describe an angle θ ($<\pi$) in time $2\sqrt{\frac{a}{3g}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{4}\right)}$. [6]

Answer <u>any one</u> question from <u>Question Nos. 16 & 17</u> : [1×6]

16. An elliptic area of eccentricity e is rotating with angular velocity ω about one latus rectum. Suddenly this latus rectum is set free and the other is fixed. Show that the new angular velocity is $\frac{\left(1-4e^2\right)}{\left(1+4e^2\right)}\omega$. [6]

17. A solid homogeneous cone of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4+\tan^2\alpha\right)$. [6]

Answer <u>any two</u> questions from <u>Question Nos. 18 to 20</u> : [2×7]

18. A particle slides from a cusp down the arc of a rough cycloid, the axis of which is vertical. Prove that its velocity at the vertex will bear to the velocity at the same point, when the cycloid is smooth, the ratio of $\left(e^{-\pi\mu}-\mu^2\right)^{1/2}:\left(1+\mu^2\right)^{1/2}$, where μ is the coefficient of friction. [7]

19. A spherical raindrop of radius 'a' centimetres falls from rest through a vertical height h, receiving throughout the motion an accumulation of condensed vapour at the rate of k grams per square centimetres per second, no vertical force but gravity acting. Show that when it reaches the ground, its radius will be $k\sqrt{\frac{2h}{g}}\left(1+\sqrt{1+\frac{ga^2}{2hk^2}}\right)$. [7]

20. A planet of mass M and periodic time T when at its greatest distance from the Sun comes into collision with a meteor of mass m, moving in the same orbit in the opposite direction with velocity v. If $\frac{m}{M}$ be small, then show that the major axis of the planet's path is reduced by $\frac{4m}{M}\frac{vT}{\pi}\sqrt{\frac{1-e}{1+e}}$. [7]

Answer <u>any one</u> question from <u>Question Nos. 21 & 22</u> : [1×6]

21. A smooth parabolic tube is placed, vertex downwards, in a vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that, in any position, the reaction of the tube is $\frac{2w(h+a)}{\rho}$, where w is the weight of the particle, ρ is the radius of curvature, 4a is the latus rectum and h is the original height of the particle above the vertex. [6]

22. A particle is moving in an elliptical orbit under inverse square law. Discuss the effect of an instantaneous change on the magnitude of the absolute acceleration 'μ', assuming that there is no change in velocity for this instantaneous change in 'μ'. [6]

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